Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-assumed

ATAR course examination 2019

Marking key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

2019/63003

Section Two: Calculator-assumed **65% (100 Marks)**

Question 7 **(6 marks)**

A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty.

(a) Write a rule for the n^{th} term of a sequence in the form $T_n = A + Bn,$ which will model this situation where T_n is the amount of water that runs out in the n^{th} minute. (2 marks)

Solution
$T_n = 84 + (n-1)(-6)$ $= 90 - 6n$
Specific behaviours
✓ states correct value for A ✓ states correct value for B

(b) How many litres run out in the seventh minute? (1 mark)

Solution
48 L
Specific behaviours
✓ states correct value

(c) How many litres have run out after eight minutes? (1 mark)

Solution
Sum of first eight terms is 504 L
Specific behaviours
✓ states correct value

(d) What is the capacity of the tank? (2 marks)

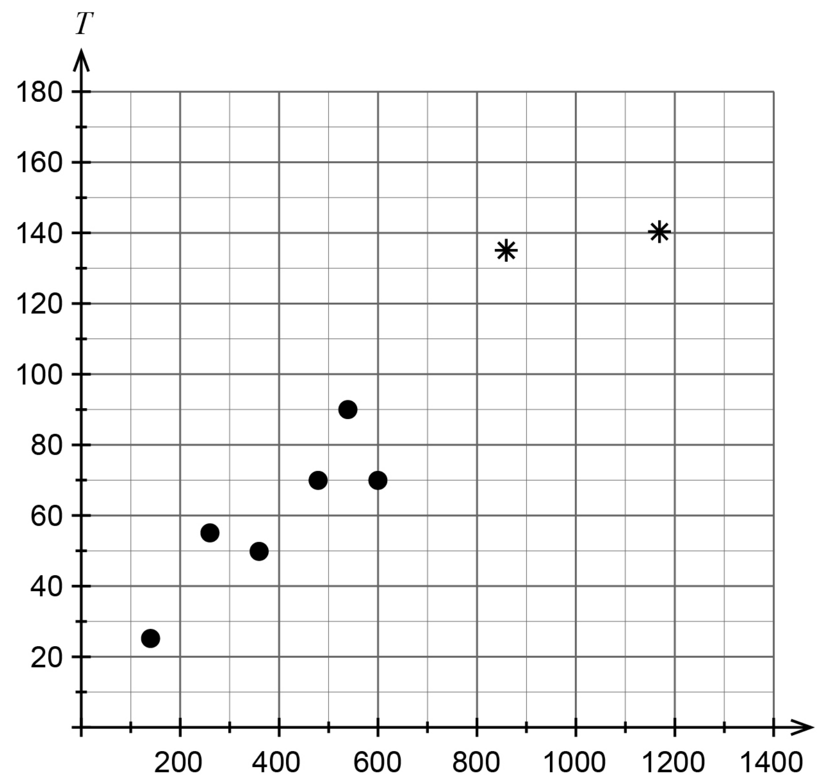
Solution
$T_{15} = 0$, Sum of first 15 terms is 630 L
Specific behaviours
✓ states that the 15^{th} term is zero ✓ states correct capacity

Question 8 **(13 marks)**

Abdul has a lawnmowing business and is investigating if there is a relationship between the size of a lawn and the length of time it takes to cut the lawn. He takes a random sample of eight customers and measures the areas of their lawns and notes the times, in minutes, it takes to mow their lawns. The results are in the table below, where A is the area of the lawn in square metres and T is the time in minutes. (Note: some values are missing.)

Customer	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
A (m^2)		260		480	540	600	860	1180
T (min)	25	55	50	70	90	70	135	140

(a) Complete the scatterplot below. (1 mark)

Solution

Specific behaviours
✓ plots both points correctly

Question 8 (continued)

(b) From the information below, determine the equation of the least-squares line in terms of A and T and state the coefficient of determination for these data. (2 marks)

Linear Reg
y = *ax*+*b*
a = 0.114691
b = 16.008241
r = 0.9510026
r^2 = 0.9044059

Solution
Equation of least-squares line is: $T = 0.115A + 16.008$ The coefficient of determination is 0.904
Specific behaviours
✓ states correct equation ✓ states correct coefficient of determination

(c) Interpret the value of the gradient of the least-squares line in the context of the question. (2 marks)

Solution
the time taken to mow the lawn increases by 0.115 minutes per square metre
Specific behaviours
✓ correctly defines an increase ✓ gives rate with correct units

(d) Given that Abdul charges $30 per hour, estimate the charge for mowing a customer's lawn with an area of 500 m^2. (2 marks)

Solution
16.008 + 0.115 × 500 = 73.508 Estimated charge = $\frac{73.508}{60} \times 30 = \$36.75$
Specific behaviours
✓ correctly calculates an estimate for T ✓ correctly calculates the charge

(e) Explain whether the estimate determined in part (d) would be valid. (2 marks)

Solution
Estimate would be valid since it is interpolation and the correlation coefficient is strong
Specific behaviours
✓ correctly explains validity ✓ explains validity with reference to either interpolation or correlation coefficient

(f) Using the least-squares line correct to three decimal places

(i) calculate the residuals for Customers B and D. (2 marks)

Solution
Residual for customer B is 9.092 Residual for customer D is –1.208
Specific behaviours
✓ correctly calculates residual for customer B ✓ correctly calculates residual for customer D

(ii) explain the significance of the sign and the size of these residuals in reference to the least-squares line. (2 marks)

Solution
The change in sign indicates the residuals are above and below the least-squares line The size indicates that the residual for D is closer to the line than the residual for customer B (or vice versa)
Specific behaviours
✓ states correct meaning of residual sign ✓ states correct meaning of residual size

Question 9 **(9 marks)**

Corbie and Grant are investigating the possibility of purchasing their own apartment. They will need to open a bank account to save for a deposit. They make an initial contribution of $7000 into the account and add a further $800 at the end of each month. The account has an interest rate of 2.6% per annum, compounded monthly.

(a) How much would be in the account at the end of two years? (3 marks)

Solution
N = 24 I = 2.6 PV = –7000 PMT = –800 FV = $27 059.30 P/Y = 12 C/Y = 12 **or** $T_{n+1} = T_n\left(1+\frac{2.6}{1200}\right)+800,\ T_0 = 7000$ $T_{24} = 27\,059.30$
Specific behaviours
✓ states correct value for N ✓ states correct values for PV and PMT (both negative or both positive) ✓ determines correct future value

Corbie and Grant are willing to pay $280 000 for the apartment and will require an additional $22 000 in fees associated with the purchase. A deposit of 20% of the total cost will be needed in order to qualify for a bank loan.

(b) (i) Show that the required deposit is $60 400. (1 mark)

Solution
20% of (280 000 + 22 000) = $60 400
Specific behaviours
✓ shows how to calculate the deposit of $60400

(ii) How long would it take to save enough for the deposit? (2 marks)

Solution
I = 2.6 PV = –7000 PMT = –800 FV = 60 400 N = 61.3 months Therefore it would take 62 months. P/Y = 12 C/Y = 12
Specific behaviours
✓ Uses correct FV in calculations ✓ determines correct time

(c) If, at the end of two years, their parents agree to give them a total of $10 000 as a reward for their dedicated saving effort, determine the minimum monthly contribution Corbie and Grant will need to make if they are to have enough for the full deposit after four years. (3 marks)

Solution
N = 24 I = 2.6 PV = –37 059.30 FV = 60 400 PMT = $868.22 P/Y = 12 C/Y = 12
Specific behaviours
✓ states correct value for PV ✓ states correct value for N ✓ determines correct payment

Question 10 **(7 marks)**

Ruby Ducks Coffee shops commenced operations in 1992 and had 15 stores open by the end of the year. They have been so successful over the years that the number of stores worldwide has continued to grow exponentially since then. The number of shops operating, T, at the end of 2017 was 22 579 and at the end of 2018 was 30 256.

The number of shops operating at the end of n years can be represented by the recursive rule $T_n = 1.34\,T_{n-1},\ T_1 = 15$.

(a) Show mathematically that the common ratio is approximately 1.34. (1 mark)

Solution
$r = \dfrac{30\,256}{22\,579} \approx 1.34$
Specific behaviours
✓ shows correct calculation of the ratio

(b) Write the rule for the n^{th} term of this sequence. (1 mark)

Solution
$T_n = 15 \times 1.34^{n-1}$ or $T_n = 11.19 \times 1.34^{n}$
Specific behaviours
✓ states correct rule

(c) Determine the first year in which there is likely to be over 200 000 Ruby Ducks Coffee shops. (2 marks)

Solution
After 33 years there are approximately 234 719 shops Therefore, the first year is 2025
Specific behaviours
✓ determines correct number of years ✓ determines correct year

Typically, each store has twelve employees working during the day across different shifts. Each employee earns, on average, $114.80 per day.

(d) Calculate the total daily wages for all stores at the beginning of 2012. (3 marks)

Solution
Stores ≈ 3900 Employees $= 3900 \times 12 = 46\,800$ Daily wages $= 46\,800 \times 114.80 = \$5\,372\,640$
Specific behaviours
✓ correctly determines the number of stores ✓ correctly determines the number of employees ✓ correctly determines the total wages

Question 11 **(13 marks)**

Data for the total occupancy of rooms for each season of the year at a Perth hotel is shown below.

n	Year	Season	Total rooms occupied	Seasonal mean	4-point centred moving average	Total rooms occupied as a percentage of seasonal mean
1	2015/16	Spring	1770	1660.5		106.59
2		Summer	1904			**B**
3		Autumn	1591		1644.375	95.81
4		Winter	1377		1622.5	82.93
5	2016/17	Spring	1641	1610.25	1618	101.91
6		Summer	1858		1614.75	115.39
7		Autumn	1601		1602.25	99.43
8		Winter	1341		1584.75	83.28
9	2017/18	Spring	1577	1524.0	1558	103.48
10		Summer	**A**		1532.375	116.93
11		Autumn	1463		1526.875	96.00
12		Winter	1274		1525.125	83.60
13	2018/19	Spring	1600	1519.75	**C**	105.28
14		Summer	1745		1525.25	114.82
15		Autumn	1504			98.96
16		Winter	1230			80.93

(a) Calculate the value of **A**, **B** and **C**. (3 marks)

Solution
$\frac{1577+A+1463+1274}{4}=1524 \Rightarrow A=1782$ $\frac{1904}{1660.5}\times 100=B \Rightarrow B=114.66$ $\frac{\frac{1463}{2}+1274+1600+1745+\frac{1504}{2}}{4}=1525.625=C$
Specific behaviours
✓ correctly calculates the value of A ✓ correctly calculates the value of B ✓ correctly calculates the value of C

Question 11 (continued)

(b) Complete the table showing the seasonal index for each season. (1 mark)

Summer	Autumn	Winter	Spring
1.1545	**0.9755**	0.8268	1.0432

Solution
See table above
Specific behaviours
✓ correctly calculates seasonal index for Autumn

(c) Calculate the deseasonalised value for Winter 2017/18. (2 marks)

Solution
$\frac{1274}{0.8268} = 1540.88$ (1541 to nearest whole number)
Specific behaviours
✓ correctly shows division by seasonal index ✓ correctly calculates deseasonalised value

(d) Comment on the effect the seasonal index had on the value found in part (c). (1 mark)

Solution
Increased value to underlying trend
Specific behaviours
✓ states correct reason

(e) The least-squares line using deseasonalised data is $R = -12.071n + 1681.25$. Use this line to predict the total number of rooms occupied during Spring 2020/21. (2 marks)

Solution
$n = 21$, $R = 1427.8$ Prediction is 1427.8 × 1.0432 = 1489.4 (1489 to nearest whole number)
Specific behaviours
✓ uses correct value of n ✓ calculates the predicted total number of rooms occupied

When a prediction was made for Spring 2020/21, using the least-squares line based on the 4-point centred moving averages, the answer was 1481.

(f) Explain why this is different from the answer obtained in part (e). (1 mark)

Solution
Different method used for smoothing
Specific behaviours
✓ states correct reason

The manager of the hotel attended a meeting with the owners of the hotel. She explained to the owners that the reduction in occupancy was due to the downturn in the Western Australian economy in recent years.

(g) Comment on the statement made by the hotel manager. (2 marks)

Solution
Cause not established Comment not appropriate
Specific behaviours
✓ states the cause in not established ✓ states the comment is not appropriate

(h) What practical advice, in the context of the question, would you give to the manager of the hotel? (1 mark)

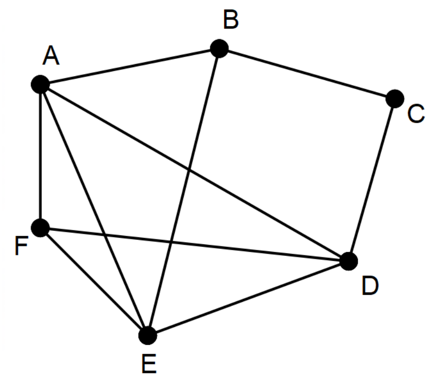
Solution
Drop room rate Advertise etc…
Specific behaviours
✓ gives a valid reason

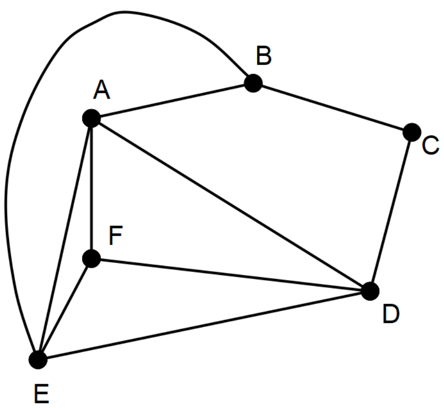
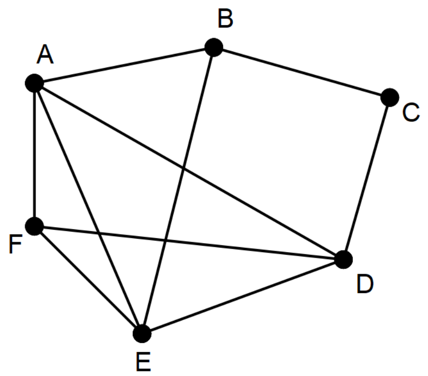
Question 12 **(6 marks)**

Jake, a park ranger, is giving a presentation at a National Park and Wildlife Conference on possible designs for a new park. Unfortunately, Jake made mathematical errors in his presentation about the paths (represented by edges) and shelter huts (represented by vertices) in the park.

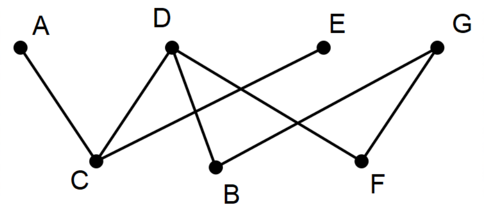
(a) For each of the following statements, the graph drawn by Jake was incorrect. Redraw the graph to match the statement correctly.

(i) This park plan has been drawn as a connected planar graph containing six vertices. (2 marks)



Solution
For example, OR
 
(the original shape is a connected planar graph)
Specific behaviours
✓ the graph is drawn with no two edges crossing ✓ the graph is drawn with all connections OR ✓✓correctly draws or states that the original is a connected planar graph

(ii) This park plan has been drawn as a bipartite graph. (3 marks)

Solution

Specific behaviours
✓ correctly arranges the groups ✓ correctly joins at least 4 vertices ✓ correctly joins all vertices

Jake also makes the following incorrect statement in his presentation. 'A park plan can be a complete graph with 21 paths and six shelter huts'.

(b) If the plan must be a complete graph with 21 paths, how many shelter huts should Jake have quoted? (1 mark)

Solution
Correct number of huts is seven
Specific behaviours
✓ states correct number of huts

Question 13 **(10 marks)**

Mehmet has saved \$3600 from wages received at a part-time job. He is keen to invest this money in an account which earns 3.65% per annum, compounded monthly.

Over the next three years, Mehmet plans to continue working part-time and is aiming to make deposits of \$250 at the end of each month.

(a) Write a recursive relation to give the value of the investment at the end of each month. (2 marks)

Solution
$T_{n+1} = T_n + \frac{0.0365}{12} \times T_n + 250, \quad T_0 = 3600$
Specific behaviours
✓ states recursive part of rule ✓ states correct starting value

(b) Mehmet hopes that this investment will double his initial savings in one year. Justify whether this is possible. (2 marks)

Solution
At the end of the 12th month Mehmet has only saved \$6784.32, therefore he does not double his money in one year.
Specific behaviours
✓ correctly states that Mehmet does not save \$7200 in one year ✓ correctly justifies this decision

(c) Determine the total amount of interest Mehmet would receive after three years. (3 marks)

Solution
Value $=13\,511.92$ Deposits $=3600+36\times 250=12\,600$ Interest $=13\,511.92-12600=\$911.92$
Specific behaviours
✓ correctly determines final value ✓ correctly determines total of all deposits ✓ correctly determines interest received

Unfortunately, after two years, Mehmet's working hours are reduced and he is only able to deposit $120 at the end of each month.

(d) By how much would this reduce the value of his investment by the end of the three years? (3 marks)

Solution
Value after 24 months $=\$10\,086.83$ Value after next 12 months $=\$11\,925.56$ Difference $=13\,511.92-11\,925.56=\$1586.36$ i.e. $1586.36 less
Specific behaviours
✓ correctly determines value after two years ✓ correctly determines value after next 12 months ✓ correctly determines the difference between future values

Question 14 **(10 marks)**

The table below contains data provided by the Australian Bureau of Statistics. It shows the number of households with and without internet access from 2014–2017. All values are in thousands of households.

State/territory	2014–15			2016–17		
	Households with internet access '000	Households without internet access '000	Total '000	Households with internet access '000	Households without internet access '000	Total '000
New South Wales	2407.9	414.5	2822.4	2439.9	421.8	2861.7
Victoria	1934.2	305.1	**A**	2008.2	305.8	2314.0
Queensland	1552.4	248.5	1800.9	1591.9	249.8	1841.7
South Australia	565.1	121.4	686.5	575.5	**B**	696.6
Western Australia	843.6	113.0	956.6	859.7	112.6	972.3
Tasmania	172.0	38.7	210.7	177.7	36.2	213.9
Northern Territory	58.1	6.3	64.4	57.6	7.3	64.9
Australian Capital Territory	137.2	9.0	146.2	140.1	9.7	149.8
Total	7670.5	1256.5	8927.0	7850.6	1264.3	9114.9

Internet access

(a) (i) Determine the value of **A** and **B** in the table above. (2 marks)

Solution
A = 1934.2 + 305.1 = 2239.3 **B** = 696.6 – 575.5 = 121.1
Specific behaviours
✓ correctly determines **A** ✓ correctly determines **B**

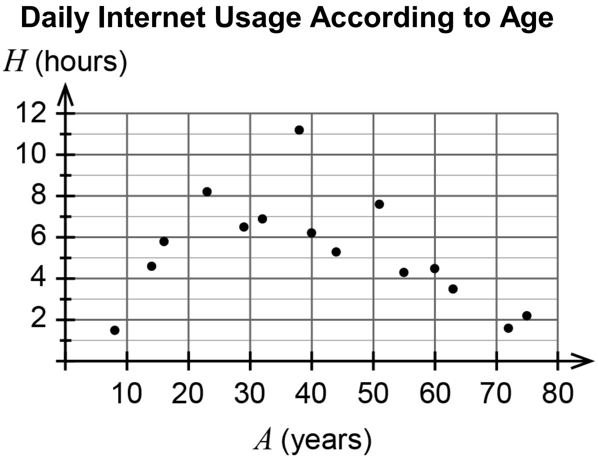
(ii) Compare the percentages, correct to two decimal places, of households with internet access in New South Wales between 2014–15 and 2016–17. Comment on your results. (3 marks)

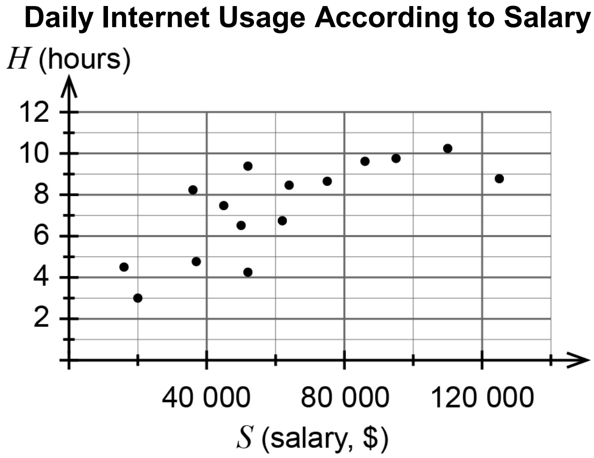
Solution
2014–15: $\frac{2407.9}{2822.4} = 85.31\%$, 2016–17: $\frac{2439.9}{2861.7} = 85.26\%$ There is a slight decrease in the percentage of households with internet access from 2014–15 to 2016–17.
Specific behaviours
✓ calculates the correct percentages ✓ correctly rounds percentages to two decimal places ✓ states there is a small drop in the percentage of households with internet access from 2014–15 to 2016–17

(iii) What is the difference in the data for households with internet access for the Northern Territory over the time period shown, compared to other States and Territories? (1 mark)

Solution
The number of households with internet access decreases from 2014–15 to 2016–17.
Specific behaviours
✓ correctly states how they are different

An internet service provider from Tasmania wanted to determine whether a person's age, A, or salary, S, affected the number of hours, H, of internet usage per day. The graphs below each show the recorded data for people surveyed.





(b) (i) Describe the association between a person's salary and the number of hours of internet usage per day, in terms of direction and form. (2 marks)

Solution
Positive and linear
Specific behaviours
✓ states correct direction ✓ correctly identifies a linear form

Question 14 (continued)

(ii) The internet service provider calculated the correlation coefficient for the data contained in each graph. The values they calculated are contained in the following list.

–1.25, –0.95, –0.75, –0.3, 0.1, 0.3, 0.75, 0.95, 1.25

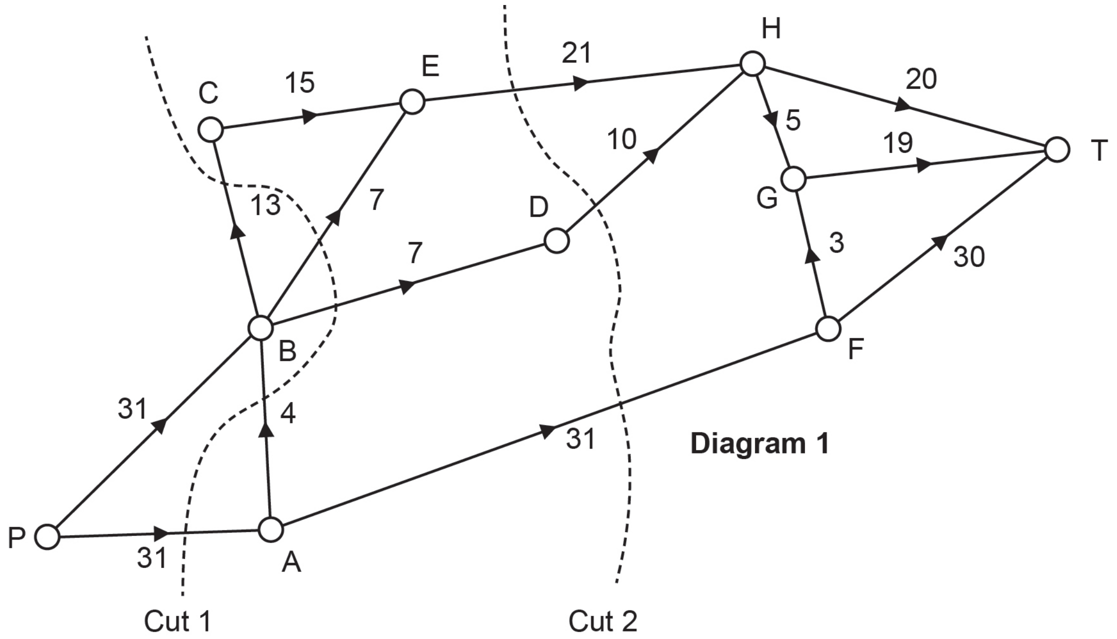
Choose the best estimate from the list for each of the graphs shown above.

(2 marks)

Solution	
Graph	**Correlation coefficient**
Daily internet usage according to age	–0.3
Daily internet usage according to salary	0.75
Specific behaviours	
✓ chooses correct correlation coefficient for internet usage according to age ✓ chooses correct correlation coefficient for internet usage according to salary	

Question 15 **(11 marks)**

The directed network below shows the maximum available capacity for transferring power between different sub-stations on a small island. The number on each edge gives the capacity in kilovolts (kV).



(a) State the capacity of each cut in Diagram 1. (2 marks)

Solution
Cut 1 = 58, Cut 2 = 62
Specific behaviours
✓ states correct value of cut 1 ✓ states correct value of cut 2

Question 15 (continued)

Diagram 2 shows a possible flow through the same network.

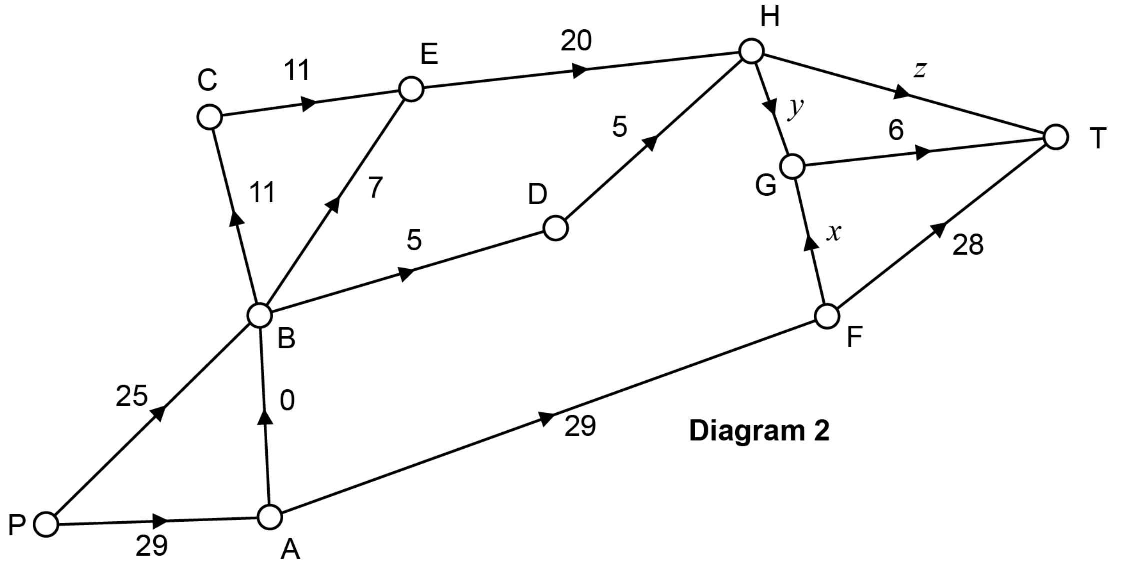


Diagram 2

(b) Determine the initial flow in Diagram 2. (1 mark)

Solution
25 + 29 = 54 which is the flow out of the source
Specific behaviours
✓ correctly shows that the initial flow is 54 kV

(c) Calculate the value of x, y and z in Diagram 2. (3 marks)

Solution
$x = 1$, $y = 5$, $z = 20$
Specific behaviours
✓ states correct value of x ✓ states correct value of y ✓ states correct value of z

(d) Determine the maximum flow for the original network (Diagram 1). (2 marks)

Solution
Minimum cut through HT, HG and AF on Diagram 1 **or** PAFT 30, PBCEHT 13, PBDHGT 5, PBEHT 7, PAFGT 1 **or** PAFT on Diagram 2 can increase by two Maximum flow = 56
Specific behaviours
✓ correctly chooses a suitable method to determine the maximum flow ✓ correctly determines maximum flow

Engineers wish to increase the maximum capacity to sub-station T. They propose to add a new transmission line from E to T of capacity 3 kV **or** a new transmission line from D to G of capacity 3 kV.

(e) Determine which of these proposals will increase the maximum capacity to sub-station T. Justify your answer. (3 marks)

Solution
ET will increase the flow by 2 (PBCET) DG will increase the flow by 2 (PBDGT) **or** cut 1 = minimum cut with edge ET or DG, i.e. an increase of 2 Therefore, either proposal will increase the flow by 2
Specific behaviours
✓ correctly shows that ET will increase the flow ✓ correctly shows that DG will increase the flow ✓ correctly justifies that either will increase the maximum capacity

Question 16 **(7 marks)**

The table below records the altitude (metres above sea level), latitude (° S) and mean maximum temperature (°C) during January for eight cities in the southern hemisphere.

Altitude (A)	Latitude (L)	Mean maximum temperature (T)
15	31.95	25
20	43.53	20
24	42.88	18
314	45.03	16
8	6.18	28
154	12.05	26
37	12.46	29
8	34.60	25

Comparing altitude and the mean maximum temperature, it was determined that the least-squares line for these data was $T = -0.022A + 24.97$ and $r_{AT} = -0.50$.

(a) Determine the coefficient of determination for altitude and the mean maximum temperature and interpret this value. (2 marks)

Solution
$r^2 = 0.25$ Approximately 25% of the variation in temperature can be explained by the variation in altitude.
Specific behaviours
✓ correctly determines the coefficient of determination ✓ gives correct description of its meaning

(b) Determine the equation of the least-squares line for comparing latitude and the mean maximum temperature and state the correlation coefficient. (2 marks)

Solution
$T = -0.264L + 30.94$ $r = -0.88$
Specific behaviours
✓ correctly states equation of least-squares line ✓ correctly states correlation coefficient

Rio de Janeiro has a latitude of 22.93° S and an altitude of 9 metres.

(c) Use the two least-squares lines above to predict the mean maximum temperature in January for Rio de Janeiro. Which prediction is more valid? Justify your choice.

(3 marks)

Solution
$T = -0.264 \times 22.93 + 30.94 = 24.89$ $T = -0.022 \times 9 + 24.97 = 24.77$ The prediction using latitude is more valid as the correlation coefficient is much stronger.
Specific behaviours
✓ correctly determines a prediction using latitude and altitude ✓ correctly states that the prediction using latitude is more valid ✓ correctly explains that latitude has a stronger correlation coefficient

Question 17 **(8 marks)**

Joel has set up a special investment fund that has a current balance of $350 000. He contributes 7.5% of his monthly income to the investment and has an overseas pension which contributes a further $355 per month. The investment fund has an interest rate of 6.5% per annum, compounded monthly. Joel's annual salary is $101 000 and he has just turned 60 years of age.

(a) Calculate Joel's total monthly contribution to the fund. (2 marks)

Solution
Monthly amount = $0.075 \times \dfrac{101000}{12} + 355 = \$986.25$
Specific behaviours
✓ calculates correct monthly income contribution ✓ calculates correct total monthly contribution

(b) Calculate the lump sum that he could receive if he retires on his 67th birthday. (2 marks)

Solution
N = 12 × 7, I = 6.5, PV = 350 000, PMT = 986.25, P/Y = C/Y = 12 FV = $655 539.45
Specific behaviours
✓ correctly uses positive (or negative) values for both PV and PMT ✓ calculates correct lump sum

Joel retires at 67 and wants to use his lump sum payment to set up a regular income. He decides to look at two options that offer monthly payments.

Option 1: A reducing balance annuity at 7% per annum, compounded monthly.
Option 2: A perpetuity at 7.5% per annum, compounded monthly.

(c) Calculate his maximum monthly income for the next 20 years using Option 1. (2 marks)

Solution
N = 12 × 20, I = 7, PV = 655 539.45, FV = 0, P/Y = C/Y = 12 PMT = $5082.39
Specific behaviours
✓ uses correct values for N and FV ✓ calculates correct monthly income

(d) Calculate his monthly income using Option 2. (2 marks)

Solution
N = (any positive integer), I = 7.5, PV = 655 539.45, FV = –655 539.45, P/Y = C/Y = 12 PMT = $4097.12
Specific behaviours
✓ uses correct values for PV and FV ✓ calculates correct monthly income

ACKNOWLEDGEMENTS

Question 14 Adapted from: Australian Bureau of Statistics. (2018, March 28). *8146.0 - Household Use of Information Technology, Australia, 2016-17* and *8146.0 - Household Use of Information Technology, Australia, 2014-15.* Retrieved May, 2019, from http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/8146.02016-17?OpenDocumentbureau
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